By following out the method indicated above we get

$$V=\tfrac{8}{3}R^3[(1+e^2)E(e)-(1-e^2)F(e)].$$

Corollary 2. By Prof. Henry Heaton, Atlantic, Iowa.

If $R=r$, $V=1/\sin\alpha\int_0^{R-\frac{1}{2}c}\sqrt{R^2-(x-\frac{1}{2}c)^2}\sqrt{R^2-(x+\frac{1}{2}c)^2}dx$, where $x+\frac{1}{2}c$ is the distance from axis of cylinder to the plane.

Put $x=(R-\frac{1}{2}c)\sin\theta$, and $(R-\frac{1}{2}c)/(R+\frac{1}{2}c)=e$.

$$\text{Then}\quad V=(2/\sin\alpha)(R-\tfrac{1}{2}c)^2(R+\tfrac{1}{2}c)\int_0^{\frac{1}{2}\pi}\cos^2\theta\sqrt{1-e^2\sin^2\theta}\,d\theta$$

$$=(2/\sin\alpha)(R+\tfrac{1}{2}c)^3[(1+e^2)E(e,\tfrac{1}{2}\pi)-(1-e^2)F(e,\tfrac{1}{2}\pi)].$$

Corollary 3. By Prof. G. B. M. Zerr, Lebanon, Va., and Prof. C. W. M. Black, Wilbraham, Mass.

If $R=r$ and $c=0$, $V=(16r^3)/(3\sin\alpha)$.

II. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.

Let $x^2+z^2=R^2$, be the equation to the cylinder.

Then $(x\cos\alpha+y\sin\alpha)^2+(z-c)^2=R^2$, is the equation to the auger hole.

The limits of y are,

$$y=\frac{\sqrt{R^2-(z-c)^2}-x\cos\alpha}{\sin\alpha}\text{ and }y=-\frac{\sqrt{R^2-(z-c)^2}+x\cos\alpha}{\sin\alpha}.$$

$$\therefore\ V=(4/\sin\alpha)\int_0^R\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}}\sqrt{R^2-(z-c)^2}\,dxdz$$

$$=(2/\sin\alpha)\int_0^R\Bigg[\left(\sqrt{R^2-x^2}-c\right)\sqrt{R^2-(\sqrt{R^2-x^2}-c)^2}$$

$$+R^2\sin^{-1}\left(\frac{\sqrt{R^2-x^2}-c}{R}\right)+\left(\sqrt{R^2-x^2}+c\right)\sqrt{R^2-(\sqrt{R^2-x^2}+c)^2}$$

$$+R^2\sin^{-1}\left(\frac{\sqrt{R^2-x^2}+c}{R}\right)\Bigg]dx.$$

This does not appear to be easy to reduce.

64. Proposed by E. S. LOOMIS, A. M., Ph. D., Professor of Mathematics in Cleveland West High School, Berea, Ohio.

Find the volume and surface generated by revolving about Y, the catenary

$$y=\tfrac{1}{2}a(e^{x/a}+e^{-x/a}),\text{ from }x=0\text{ to }x=a.$$

[*Osborne's Calculus*, page 255, example 8].

I. Solution by S. ELMER SLOCUM, Professor of Mathematics Union College, Schenectady, New York; J. M. BANDY, A. M., Professor of Mathematics in Trinity College, Trinity, N. C.; COOPER D. SCHMITT, A. M., Professor of Mathematics in the University of Tennessee, Knoxville, Tenn.; and B. F. SINE, Principal of Rock Enon High School, Rock Enon Springs, Va.

$$V=\pi\int_o^a x^2dy. \quad y=\tfrac{1}{2}a(e^{x/a}+e^{-x/a}). \quad \therefore\ dy=\tfrac{1}{2}(e^{x/a}-e^{-x/a})dx.$$

$$\therefore\ V=\tfrac{1}{2}\pi\int_o^a x^2(e^{x/a}-e^{-x/a})dx.$$

Integrating by parts,

$$V=\tfrac{1}{2}\pi\Big[(ax^2e^{x/a}-2a^2xe^{x/a}+2a^3e^{x/a})+(ax^2e^{-x/a}+2a^2xe^{-x/a}+2a^3e^{-x/a})\Big]_o^a.$$

$$\therefore\ V=\tfrac{1}{2}\pi[a^3e+5a^3e^{-1}-4a^3]=\tfrac{1}{2}\pi a^3[e+5e^{-1}-4].$$

$$S=2\pi\int xds=2\pi\Big[xs-\int sdx\Big],$$

where s=length of arc of catenary, $=\tfrac{1}{2}a(e^{x/a}-e^{-x/a})$.

$$\therefore\ S=2\pi\Big[\tfrac{1}{2}ax(e^{x/a}-e^{-x/a})-\tfrac{1}{2}a\int(e^{x/a}-e^{-x/a})dx\Big]\Big/_o^a$$

$$\text{or } S=2\pi\Big[\tfrac{1}{2}ax(e^{x/a}-e^{-x/a})-\tfrac{1}{2}a^2(e^{x/a}+e^{-x/a})\Big]\Big/_o^a.$$

$$\therefore\ S=2\pi[\tfrac{1}{2}a^2(e-1/e)-\tfrac{1}{2}a^2(e+1/e)+\tfrac{1}{2}2a^2]. \quad S=2\pi a^2(1-e^{-1}).$$

II. Solution by J. OWEN MAHONEY, B. E., M. Sc., Graduate Fellow and Assistant in Mathematics, Vanderbilt University, Nashville, Tenn.; and HENRY HEATON, M. Sc., Atlantic, Iowa.

(a). The volume is given by the equation

$$V=\pi\int_o^a x^2dx=\tfrac{1}{2}\pi\int_o^a x^2(e^{x/a}-e^{-x/a})dx.$$

Let $x=ay$, then the limits of y are 0 and 1.

Hence $V=\pi\int_o^a x^2dy=\tfrac{1}{2}\pi a^3\int_0^1 y^2(e^y-e^{-y})dy.$

Integrating by parts, we find,

$$\int y^2e^ydy=(y^2-2y+2)e^y, \text{ and } \int y^2e^{-y}dy=-(y^2+2y+2(e^{-y}.$$

Therefore $V=\dfrac{\pi a^3}{2}\Big[(y^2-2y+2)e^y+(y^2+2y+2)e^{-y}\Big]_0^1$

$$=(\pi a^3/2)(e+5e^{-1}-4).$$

(b). The surface is given by

$$S=2\pi\int_o^a x\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}dx=\pi\int_o^a x(e^{x/a}+e^{-x/a})dx$$

$$=\pi a\Big[e^{x/a}(x-a)-e^{-x/a}(x+a)\Big]_o^a=2\pi a^2(1-e^{-1}).$$